SECTION I.—AEROLOGY.

PRINCIPIA ATMOSPHERICA: A STUDY OF THE CIRCULATION OF THE ATMOSPHERE.[1]

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(An address delivered at the request of the council before the Royal Society of Edinburgh, on December 1, 1913.)

[Read December 1, 1913. MS. received December 12, 1913.]

Synopsis.

SECTION I.—AXIOMS OR LAWS OF ATMOSPHERIC MOTION.

1. *The Law of the Relation of Motion to Pressure.*

In the upper layers of the atmosphere, the steady horizontal motion of the air at any level is along the horizontal section of the isobaric surfaces at that level, and the velocity is inversely proportional to the separation of the isobaric lines in the level of the section.

2. *The law of the Computation of Pressure and of the Application of the Gaseous Laws.*

The pressure at any point in the atmosphere and at any instant is the weight of the column of air which stands upon one unit of horizontal area containing the point. The numerical values of pressure, temperature, and density at any point of the atmosphere are therefore related by the usual formulæ for the gaseous laws.

3. *The Law of Convection.*

Convection in the atmosphere is the descent of colder air in contiguity with air relatively warmer.

4. *The Law of the Limit of Convection.*

Convection in the atmosphere is limited to that portion of it, called the troposphere, in which there exists a sensible fall of temperature with height. The upper layer of the atmosphere, in which there is no sensible fall of temperature with height and therefore no convection, is called the stratosphere.

5. *The Law of Saturation.*

The amount of water vapor contained in a given volume of air can not exceed a certain limit, which depends upon the temperature and upon nothing else.

SECTION II.—LEMMAS OR POSTULATES.

Lemma 1.—In the stratosphere, from 11 kilometers upward it is colder in the high pressure than in the low pressure at the same level; and in the troposphere, from 9 kilometers downward to 1 kilometer, it is warmer in the high pressure than in the low pressure at the same level. [W. H. Dines, *M. O.*, 210*b*.]

Lemma 2.—The average horizontal circulation in the Northern Hemisphere in January between 4 kilometers and 8 kilometers consists of a figure-of-eight orbit from west to east along isobars round the pole, with lobes over the continents and bights over the ocean.

The average circulation at the surface is the resultant of the circulation at 4 kilometers combined with a circulation in the opposite direction of similar shape due to the distribution of temperature near the surface. [L. Teisserenc de Bort, *Ann. du Bureau Central Météorologique*, 1887; and W. N. Shaw, *Proc. Roy. Soc.*, vol. lxxiv, p. 20, 1904.]

SECTION III.—PROPOSITIONS.

Proposition 1.—To define the conditions for the persistence of the existing motion of the atmosphere.

Proposition 2.—To show that the rate of increase of pressure difference per kilometer of height is $34\cdot2\frac{p}{\theta}\left(\frac{\Delta\theta}{\theta}-\frac{\Delta p}{p}\right)$; and hence that the distribution of pressure in the stratosphere is the dominant factor in the circulation of the air at the surface; that the intermediate layers between 4 kilometers and 8 kilometers exert little influence upon the distribution of pressure.

Proposition 3.—To show that the wind velocity across the slope of pressure at any level is proportional to $\theta\frac{\Delta p}{p}$; and thence to show how to utilize observations of the pressure and temperature to calculate the wind velocity at any level.

Proposition 4.—To show that the wind velocity generally increases with height until the substratosphere is reached, and falls off with increase in height in the stratosphere.

Proposition 5.—To show how the distribution of pressure and temperature in the upper air can be calculated from the observations of structure represented by a sounding with a pilot balloon, and thence to account for the local distribution of rainfall when an upper current from the northwest crosses a lower current from the southwest.

Proposition 6.—To account for the average general circulation over the Northern Hemisphere in the 4-kilometer level as set out in Lemma 2.

INTRODUCTION.

Every science has two aspects or two stages in its development. In the first, the inductive stage, observations are made and compiled, and axioms or laws are laid down. In the second or deductive stage the laws are applied by syllogistic reasoning, mathematical or otherwise, to elicit conclusions which either disclose new facts or show the inevitable connection between facts already known, and, in either case, complete the claim of the study to the rank of a science.

The different sciences vary greatly in the stage of development which they present. The science of geometry has almost forgotten the origin of its own laws and axioms, and occupies itself with the most complicated deductive propositions, the forms of which are used to guide the deductions of other sciences. Biology is still in the inductive stage; no one ventures yet to predict in what form the horse will be found a million or even a thousand years hence.

These different aspects of science appeal with different force to different types of human mind. Observers are comparatively rare; true inducers, those who have the patience and the insight to arrange the facts and formulate the underlying laws are extremely rare; deducers, those who draw conclusions, not always mathematical or strictly logical, make up the balance of the human race.

Many years ago, in 1862, Dr. Alexander Buchan, in a contribution to this society which was subsequently elaborated in a volume of the results of the *Challenger* Expedition, laid the foundations of our inductive knowledge of the atmospheric circulation by a series of maps of the distribution of pressure over the surface of the globe. With great pleasure I take the opportunity afforded to me by your invitation to address you on recent developments of the science of meteorology particularly

[1] *Reprinted from* Proc. Roy. Soc. Edinb., 1913-1914, **34**, 77-112, as issued separately Mar. 23, 1914.

in the investigation of the upper air, to put before you a representation of the knowledge of the atmospheric circulation as it presents itself to my mind, arranged in the normal scientific form, with axioms which represent inductive laws, with postulates or lemmas which represent groups of observed facts, and with propositions leading to conclusions which are susceptible of verification.

SECTION I.—AXIOMS OR LAWS OF ATMOSPHERIC MOTION.

The time has arrived when it seems possible and desirable to formulate the laws and principles which can be effectively employed at the present day in the explanation of many of the recognized phenomena of the structure and circulation of the atmosphere and to illustrate their application. These laws and principles are the result of observations sometimes suggested or controlled by theory. They are of the nature of axioms or inductions, about the validity of which a good deal of discussion is possible. Into that discussion I do not now propose to enter. The axioms really depend for their justification upon their effectiveness in explaining observed facts. They are set out as follows:

1. *The law of the relation of motion to pressure.*

In the upper layers of the atmosphere, the steady horizontal motion of the air at any level is along the horizontal section of the isobaric surfaces at that level, and the velocity is inversely proportional to the separation of the isobaric lines in the level of the section.

The line of argument in favor of this law, which can not, strictly speaking, be either verified or contradicted by any available process of observation, is as follows: The condition specified in the law is the condition of kinematic equilibrium toward which all atmospheric motions tend, and have tended either since the earth began to rotate as it does now, or the atmosphere was first formed, whichever of those events is the later in time. Any deviation from the equilibrium state is by infinitesimal steps during which readjustment to the equilibrium condition has been taking place automatically. Hence any finite difference from the equilibrium state can only occur in quite exceptional conditions. Consequently if there is an ascertained difference from the equilibrium condition it requires explanation, just as the divergences from the uniformity contemplated by the First Law of Motion require explanation.

An allowance for "curvature of path" is one of the differences of which account may have to be taken. Its importance depends upon the latitude. For the half of the globe north of 30° N. and south of 30° S. it is generally negligible, but near the equator it becomes the paramount consideration in the question of the persistence of distribution. Thus rotary systems, small or large, are the only possible isobars for a synchronous chart of an equatorial region, if one were drawn. The long sweeps of "parallel isobars" with which we are concerned in this paper would be inadmissible there.

Near the surface there is always a component of motion along the gradient from high pressure to low pressure. In this region the friction due to obstacles and to the viscosity of the air prevents the steady state being reached, and in consequence the centrifugal force due to the velocity of motion is not adequate to balance the pressure.

This modification of the general principle in the case of surface air may be inferred from the fact that in all maps of the distribution of pressure and wind at the surface there is evidence of a flow across the isobars. The maps are not always conclusive, as they are for sea level and not station level; but no person of experience will doubt the general truth of the statement, which in books often takes the form of postulating convergence toward centers of low pressure and divergence from centers of high pressure.

2. *The law of the computation of pressure and of the application of the gaseous laws.*

The pressure at any point in the atmosphere and at any instant is the weight of the column of air which stands upon one unit of horizontal area containing the point.

This principle assumes that the motion of the air is so slow that the hydrostatical forces are not interfered with. Explosion or elastic wave motion would invalidate the law. It therefore assumes that the atmosphere is free from explosions and elastic wave motions, or that their effect is so small that it does not enter into meteorological calculation.

It follows that the numerical values of pressure, temperature, and density at any point of the atmosphere are related by the usual formulæ for the gaseous laws. In other words, when due allowance is made for the difference of composition in consequence of the variation in the amount of water vapor or other possible causes, the relation $p = R\theta\rho$ holds, where p, θ, ρ are the pressure, temperature (on the absolute scale), and density of the air, and R is a "constant" which is altered by an alteration in the composition of the air but not by other causes.

3. *The law of convection.*

Convection in the atmosphere is the descent of colder air in contiguity with air relatively warmer.

The law is advisedly stated in this form (although objections may be taken to it for want of strictness) because the driving power of the convective circulation comes from the excess of density of the descending portion, and the excess of density in atmospheric air is due in nearly all cases to low temperature. Differences of density might be caused by differences of pressure or by differences in the amount of moisture contained in equal volumes; but finite differences of pressure cannot persist in contiguous masses of air. The amount of water vapor in air at the ordinary temperatures with which a meteorologist has to deal is only a small fraction of the whole mass, and the colder the air is the less water vapor is required to saturate it. Consequently, although it would be possible in a physical laboratory to display a sample of air which though warmer is yet denser than another cooler sample on account of the humidity of the latter, the conditions would not easily occur in nature, and the motive power for convection would be exceedingly small. Such cases may therefore be left out of account, and we may consider that of two contiguous masses of air the colder is the denser.

The law of convection is usually stated with regard to the warmer part of the convective circulation and takes the briefer form that warm air rises. The general adoption of this briefer form is due to the fact that the warming of air at the surface is a matter of common knowledge and it occurs in the daytime, when its effects in producing a local convective circulation are often quite distinctly visible. The form which is adopted here, however, is preferable, because in any case it is the cooler and heavier air in the neighborhood which must be looked for if the true cause of the circulation is to be found, and although on the smaller scale the heavier air is not far to seek, it is not so easily identified on the scale of a meteorological chart.

Convection in the atmosphere may also be due to the variation in the gravitational acceleration due to the motion of the air with reference to the earth.

The gravitational acceleration depends partly on the statical attraction of the earth's mass and partly on the centrifugal action due to rotation. The ordinary values of the constant of gravitation assume the rotation to be that of the solid earth, and the acceleration of gravity upon air moving over the earth's surface is consequently different from that for calm air. Hence the air which forms part of a westerly wind is *specifically lighter* than air at the same temperature and pressure which is calm, and, on the other hand, air which forms part of an easterly wind is *specifically heavier*. These variations in what, contrary to the usual convention, may rightly be called the "specific gravity of the air" have not yet been generally taken into account in meteorological practice, but they are of real significance and are the subject of certain classical papers by von Helmholtz and Brillouin on the circulation of the atmosphere.

4. *The law of the limit of convection.*

Convection in the atmosphere is limited to that portion of it in which there exists a sensible fall of temperature with height.

This portion, which comprises about three-fourths of the atmosphere, is called the *troposphere* and is a layer of air about 10 kilometers thick surrounding the whole earth. It is surrounded by an outer spheroid of air comprising the remaining fourth part of the atmosphere, which is called the *stratosphere*, in which there is no sensible fall of temperature with height. The boundary between these two layers is not at a fixed height; it is apparently a flexible, and therefore deformable surface, but it is not penetrable by air.

The height of the boundary differs in different latitudes, being highest over the equator and getting gradually lower towards the poles; it differs also in different localities, being higher over an area of high pressure than over one of low pressure. The local differences are due to deformations of the boundary by the accumulation or withdrawal of air from underneath. At any place the boundary oscillates about a mean position which should be regarded as the height of the boundary of the stratosphere for the place. There is no physical reason why the boundary of the stratosphere should not be penetrated. All that is required to produce that effect is an accumulation of air warm enough to cause upward convection. All that can be said is that there is no example of the approach to such an accumulation. There are a sufficient number of examples in which there is a reversal of fall of temperature just below the stratosphere, and these show that the stratosphere has, if anything, a little to spare in the way of resistance against penetration. Hence, from the point of view of meteorological theory, we regard the stratosphere as impenetrable.

5. *The law of saturation.*

The amount of water vapor contained in a given volume of air cannot exceed a certain limit which depends upon the temperature and upon nothing else.

This is really simply a statement of Dalton's law of the saturation of a gas with the vapor of a liquid, but it is quoted here partly because it refers to the only form of variation in chemical composition to which the meteorological atmosphere is subject, and also partly in order to avoid a misapprehension that is very widespread. It is a well-known physical principle that when a vapor is condensed the "latent heat of vaporization," which, in the case of water vapor, is very large, is liberated. The statement of the principle is not complete; it should go on to say that the condensation cannot take place unless provision has been made for disposing of the heat which will be liberated. In the case of the atmosphere it is often assumed that no provision of the kind is required, and that the air will, in consequence, *be warmed* by the heat set free. Herein lies the misapprehension. Vapor of water in air will not condense unless the air is cooled, and the amount of condensation will be limited by the amount of the cooling.

It should, however, be noted that the wording of the law as here given, namely, that the limiting amount of water vapor depends upon the temperature *and upon nothing else*, implies a statement about the atmosphere about which it is necessary to be explicit. Since Dalton's law was enunciated, the researches of Aitken and others have shown that the cooling of a mass of air below the "saturation point" causes condensation only if there are nuclei upon which drops of water can form. In the absence of such nuclei, laboratory experiments have shown that condensation does not take place until the limits of saturation have been largely exceeded; "four-fold saturation" is necessary in such a case. Air without nuclei cooled below its "saturation point" is said to be supersaturated, and the statement of the law of saturation as set out implies that *supersaturation does not exist in the free air*. This is another case in which there is no physical reason to prevent anyone imagining circumstances in which supersaturation might exist; all that can be said is that no such circumstances have been demonstrated, and the ready formation of clouds at all heights seems to indicate that such circumstances are quite unlikely. Hence the meteorologist is entitled to infer, as the result of a meteorological though not of a physical law, that *condensation* in the form of cloud, or if necessary of rain, *will always accompany the reduction of temperature of the air below the point of saturation*, and the amount of condensation will depend upon the reduction of temperature and upon nothing else.[2]

These five laws express the special principles with which the meteorologist must approach the consideration of the circulation of the atmosphere, with all its complexities and its perplexities. The rest must depend upon the application of the ordinary principles of dynamics and physics to the results of observations which indicate the pressure, temperature, and density of the air in its actual condition when under consideration. It is my object in this paper not to discuss or to justify these principles, but to show how far they lead us in the explanation of some of the more general phenomena of the atmospheric circulation.

The form which has been adopted for this communication has been chosen for the purpose of drawing a distinction between the inductive, the observational, and the deductive aspects of the questions which are treated. Just as, in the cases of motion treated in text-books of dynamics, there is ample opportunity for discussion as to the form of words which shall be used for the laws of motion and the grounds for their acceptance or rejection,

[2] The supersaturation of atmospheric air is discussed in Dr. Alfred Wegener's Thermodynamik der Atmosphäre, Leipzig, J. A. Barth, 1911. Humidities, by the hair hygrometer, up to 107 per cent are cited on p. 254 of that work.

starting from the consideration that there never has been an actual example of a body free from the action of force, so, in the case of atmospheric motion, there is no lack of opportunity for the discussion of the laws as here set out, starting from the consideration that no actual case can be quoted in which we are certain that the laws are strictly obeyed. And further just as in the case of the dynamics of the heavenly bodies the whole subject is reduced to a manageable form by setting out to explain the changes of motion and their causes instead of pondering over the ultimate origin and cause of the state of motion which exists at any particular epoch, so in the study of the circulation of the atmosphere we may profitably turn our attention to the changes in the motion related to the varying distributions of pressure, and leave for the time being the endeavour to give a short answer to the question, "What is the ultimate cause of any given distribution of pressure, with its attendant atmospheric motion?"

We proceed, therefore, first to define in two lemmas the average condition of the atmosphere which we wish the reader to keep in mind, and secondly to apply the laws which have been already enunciated to make certain deductions or establish certain propositions with regard to the circulation of the atmosphere, which are set out in the synopsis.

SECTION II.—LEMMAS OR POSTULATES.

Lemma 1.

In the stratosphere from 11 kilometers upward it is colder in the high pressure than in the low pressure at the same level; and in the troposphere, from 9 kilometers downward to 1 kilometer, it is warmer in the high pressure than in the low pressure at the same level.

Proof.—Table of average values of pressure and temperature at different levels over high pressure (1031 *mb.*) and low pressure (984 *mb.*) at the surface; with pressure differences and temperature differences at each level. Compiled from the diagram and tables of W. H. Dines, F. R. S., in *Geophysical Memoirs*, No. 2, M. O. Publication, 210*b*.

TABLE 1.

Altitude.	Pressure.		Diff.	Diff.	Temperature.	
	Low.	High.	Δp.	$\Delta\theta$.	984 *mb.* low.	1031 *mb.* high.
km.	*mb.*	*mb.*	*mb.*	°*A.*	°*A.*	°*A.*
15	116	123	7	....	...	...
14	135	146	11	− 9	224	215
13	157	171	14	−11	226	215
12	183	201	18	− 8	225	217
11	212	235	23	− 4	225	221
10	247	273	26	+ 1	225	226
9	288	317	29	+ 7	226	233
8	335	366	31	+13	227	240
7	388	422	34	+15	232	247
6	449	483	34	+14	240	254
5	516	552	36	+13	248	261
4	591	628	37	+12	255	267
3	675	713	38	+ 9	263	272
2	767	807	40	+ 8	269	277
1	870	913	43	+ 4	275	279
0	984	1031	47	+ 3	279	282

Standard deviation of P_9 13.8 mb.
Standard deviation of P_8 14.1 mb.
Correlation coefficient between the variations of P_9 and P_8 from the means for the month (English ascents) 0.80.

The table which is here given summarizes the results of an important investigation by Mr. Dines into the relation between the changes of pressure at the 9-km. level and the corresponding changes at the surface. The changes which he dealt with were chronological, and I have extended the conclusion in applying it to topographical differences. This extension is justified if the places between which the differences are to be taken are sufficiently close together to be influenced by the same barometric system, and if the chronological sequence is followed in individual cases. That the latter condition is generally satisfied is shown by the high correlation coefficient between the variations at 9 km. and at the surface.

The conclusion as to the relation between temperature and pressure in the upper air, which is drawn from this table, is supported by the gradual evolution of meteorological ideas on the subject. Originally it was assumed that high pressure meant relatively dense air and low pressure relatively light air from the surface upward. Sometimes temperature and sometimes moisture was held accountable for the levity; but the view first put forward by von Hann that, in ordinary circumstances, the air over high pressure is warmer than that over low pressure has gradually developed until it may now be regarded as an accepted principle in meteorology. It is borne out by the simultaneous soundings which have occasionally been obtained from places within the same barometric system; and apparently the disturbances in the specified order are mostly confined to the lowest reaches of the atmosphere. This last point also is well illustrated by the figures of the table, which show a gradual falling off, on the average, of the temperature differences in the lowest three kilometers.

Lemma 2.

The average horizontal circulation in the Northern Hemisphere in January between 4 kilometers and 8 kilometers consists of a figure-of-eight orbit from west to east along isobars round the pole, with lobes over the continents and bights over the oceans.

The average circulation at the surface is the resultant of the circulation at 4 kilometers combined with a circulation in the opposite direction of similar shape due to the distribution of temperature near the surface.

[L. Teisserenc de Bort, *Ann. du Bureau Central Météorologique*, 1887; and W. N. Shaw, *Proc. Roy. Soc.*, vol. lxxiv. p. 20, 1904.]

This lemma is introduced in order to supply the reader with a suitable general picture of the atmospheric circulation in the upper air, and the modification which it must undergo in the lowest layers in consequence of the distribution of temperature near the surface. As will be seen from Proposition 2, which follows, the similarity of pressure-distribution at all heights depends upon the equality of $\Delta\theta/\theta$ and $\Delta p/p$. Consequently, a circulation along parallels of latitude from west to east in which the air nearer the poles is the colder is a circulation which may remain practically identical at all heights, and is suggestive of durability and persistence.

The distribution of pressure at the 4-km. level given by M. Teisserenc de Bort suggests that the actual circulation in the upper air is not a circulation along parallels of latitude, but yet is an approximation thereto, being something intermediate between a circle and a figure-of-eight.

That the circulation at the 4-km. level remains of the same general character up to the 8-km. level is suggested by the fact that in those regions distribution of temperature is such as to cause very little change in pressure differences, in accordance with the formula of Proposition 2.

It may be remarked that the distribution was calculated by M. L. Teisserenc de Bort from the distribution of pressure and temperature at the surface, and is subject to two uncertainties, first, the reduction of the original pressure readings to sea level; and, secondly, their further reduction to the 4-km. level. The uncertainties arise from the uncertainty in the values of the temperature of the air "below the ground" in the reduction to sea level and above the ground in the reduction to the 4-km. level. To a certain extent these two uncertainties compensate each other in the important features of the result, and the conclusion as to the circulation at which M. Teisserenc de Bort had arrived is supported by the results of Hildebrandsson's discussion of the international cloud observations (see Hildebrandsson and Teisserenc de Bort, *Les Bases de la météorologie dynamique,* vol. ii, Gauthier-Villars, Paris), and by other considerations of a more general character.

The statements of these two lemmas are based upon observation and are therefore liable to modification or correction in detail as the results of observation become more conclusive. They are, however, sufficiently well established to justify their use in the further consideration of meteorological problems.

SECTION III.—PROPOSITIONS.

We now proceed to the consideration of the propositions which are set out in the synopsis. I shall deal in detail with only three of the propositions, numbered 1, 5, and 6, respectively, because the remaining three, numbered 2, 3, and 4, have already been dealt with in a paper communicated to the Scottish Meteorological Society, with the title of "The Calculus of the Upper Air, and the Results of the British Soundings in the International Week of May 5–11, 1913." The paper is published in the *Journal* of the society for 1913.

Proposition 1.—The conditions necessary to maintain a steady atmospheric current.

The conditions which must be complied with if a steady current is to be persistently maintained must satisfy the first law, the law of relation of motion to pressure.

The law prescribes that the velocity V is related to the pressure gradient γ, density ρ, latitude λ, and the angular velocity of the earth's rotation ω, by the equation

$$V=\gamma/(2\omega\rho \sin \lambda).$$

Provided that the latitude λ remains constant during the persistence of the current this condition presents no difficulty; the flow will always be determined by the distance apart of the isobars, but the auxiliary condition that the current shall not change its latitude implies that the isobars are parallel to the circles of latitude. Hence we may infer that, neglecting a very small correction for curvature, a circulation round the pole along isobars parallel to the circles of latitude is a "steady" circulation which will be persistently maintained. The only forces which will interfere with it are frictional forces due to the relative motion of adjacent layers of air, and, except in the immediate neighborhood of the ground where friction is aided by turbulent motion, these are extremely small. Hence a west-to-east circulation or an east-to-west circulation in the upper air once steady will remain so, unless it is disturbed by changes of pressure distribution.

But, on the contrary, when the air movement is from south to north or from north to south, or has any component which gives a motion across the circles of latitude, a change in $\sin \lambda$ has to be dealt with.

Motion from south to north.

We propose to deal first with a current moving from south to north. We shall suppose the current to be uniform over the section *from the one-kilometer level upwards.* We leave out the lowest kilometer because we know that it is disturbed by quasi-frictional forces at the surface.

In this case the value of $\sin \lambda$ is increasing, and therefore greater pressure difference is required to get the same quantity of air through the same section. But the pressure difference is limited by the isobars, which are by hypothesis supposed steady. Any convergence of the isobars themselves provides its own remedy, because the gradient velocity is inversely proportional to the distance. We have therefore only to deal with the change in $\sin \lambda$ in the formula

$$V=\gamma/(2\omega\rho \sin \lambda).$$

Let L be the width of the current, and H its depth; then the flow over the whole section $L\times H$ is LHV; and by the equation of continuity this must be constant as the stream flows northward.

Now

$$LHV=\frac{HL\gamma}{2\omega\rho \sin \lambda},$$

and $L\gamma$ is the pressure-difference, Δp, between the two sides of the current. LHV is constant; hence, differentiating, we get

$$0=\frac{\partial H}{H}-\frac{\partial \rho}{\rho}-\frac{\partial \sin \lambda}{\sin \lambda}$$

or

$$\frac{\partial H}{H}=\frac{\partial \rho}{\rho}+\cot \lambda\, \partial\lambda.$$

Now ρ can only alter by variation of pressure, temperature, or composition; change of pressure is ruled out because the motion is along isobars; change of temperature will be very slight because there is no change of pressure, and there are no other causes of any appreciable change of temperature; and change of composition can only occur in consequence of condensation. By Law 5, in the absence of change of temperature no change of composition will occur. Hence

$$\partial\rho/\rho=0,$$

and

$$\frac{\partial H}{H}=\cot \lambda\partial\lambda.$$

In other words, the thickness of the moving layer must increase fractionally by the amount $\cot\lambda\delta\lambda$ for the change of latitude $\delta\lambda$. If latitude is expressed in degrees and not in circular measure as differentiation supposes, we must introduce the factor $\frac{\pi}{180}$, and thus the formula becomes

$$\frac{1}{H}\cdot\frac{dH}{d\lambda}=0\cdot 0175 \cot \lambda.$$

Hence, in order that a current may persist over any stretch from south to north, *it is necessary that the thickness of the moving layer should increase fractionally to the extent of $0\cdot 0175 \cot \lambda$ for every degree of latitude which it crosses.*

We have assumed the layer to be unlimited above, and limited below by the one-kilometer level. To provide for the additional air by increasing the height above the selected base-level would result in altering the pressure: that mode of operation is therefore excluded by the condition of maintenance of the current as steady. Consequently we must suppose the additional thickness to be provided by encroachment upon the lowest kilometer: that region is already supposed to be occupied by an extension of the current which is disturbed by surface friction; hence, *unless there is a continual flow-off of air from below the one-kilometer level, the steady state can not be maintained.*

The south-to-north current implies a high pressure on the eastern side and a low pressure on the western side, and near the surface there is a component of flow from high to low across the isobars. Hence we may suppose a case in which the northward-flowing current is maintained steady by the flow-off from east to west in the surface layer. We proceed to calculate the amount of this east-to-west current which will suffice to draw off the increase of the current above 1 kilometer.

We suppose, for the purpose of calculation, that the east-to-west component is uniform over the lowest half kilometer of the western section. The fractional increase of thickness in the upper layer has been shown to be $0\cdot0175 \cot\lambda$ for each degree of advance northward. The increase of the thickness is the same over each elementary layer of height into which the whole thickness can be divided; consequently the air to be removed is the fraction $0\cdot0175 \cot\lambda$ of the transverse vertical section at every level. If the removal is confined to the lowest half kilometer, which contains a fraction of the atmosphere approximately one-twentieth of the whole, it follows that a fraction $20\times0\cdot0175 \cot\lambda$ of the lowest half-kilometer layer has to be removed for each degree of advance northward.

For each *meter* of advance northward, therefore, a fraction $\dfrac{20\times0\cdot0175\cot\lambda}{111\cdot1\times10^5}$ of the lowest half-kilometer layer has to be removed; and, similarly, for each *meter per second* of the wind velocity from south to north a fraction $\dfrac{20\times0\cdot0175\cot\lambda}{111\cdot1\times10^5}$ must be removed *every second.*

Suppose that the breadth of the advancing current which is supposed to be maintained steady is L kilometers, the westerly flow at the western end of the lowest half kilometer must carry away air at the rate of $\dfrac{20\times0\cdot0175\cot\lambda}{111\cdot1\times10^5}\times L$ kilometers per second, or there must be a cross component of wind there amounting to $\dfrac{20\times0\cdot0175\cot\lambda}{111\cdot1}\times L$ meters per second.

If the cross wind be referred to the width of a current expressed in degrees of longitude at the latitude λ, and if l be the width of the current in degrees, we get

$$L=111\cdot1\cos\lambda\, l.$$

Whence it follows that in order to maintain a south-to-north current of V meters per second there must be a cross wind leaving the lowest half kilometer of

$$0\cdot35\cos^2\lambda\,\frac{lV \text{ meters per second}}{\sin\lambda}.$$

We have supposed the drainage to take place entirely in the lowest half kilometer, which represents one-twentieth of the atmosphere. The same result might be produced by a distributed crossflow throughout the western vertical section of the moving air of $0\cdot0175\,\dfrac{\cos^2\lambda}{\sin\lambda}lV$ meters per second.

We may therefore sum up the conclusion as follows: *In order that a current across circles of latitude from south to north with a breadth of l degrees of longitude may persist unaltered at any level, it is necessary that air should be drawn away from the moving air at that level to the extent of* $0\cdot0175\,\dfrac{\cos^2\lambda}{\sin\lambda}lV$ *meters per second.*

The use of the surface layer, to draw off the excess of air which would otherwise prevent the persistence of a current across circles of latitude, is quite appropriate in the case of currents with a south-to-north component. According to the rider to Law 1, such a current certainly exists, and it only requires its magnitude to be adjusted in order that persistence may be secured. For a current extending over 10° of longitude in latitude 45° the cross component at the extreme west of the lowest half kilometer would have to be two and a half times the steady south wind above, and that hardly occurs in practice; but there are a variety of ways of accounting for any discrepancy between the calculated and observed cross wind in case the south-to-north current is actually maintained. Hence the diagram, figure 1, representing the conditions for maintenance of a south wind across a section of 5° of longitude is not unreasonable.

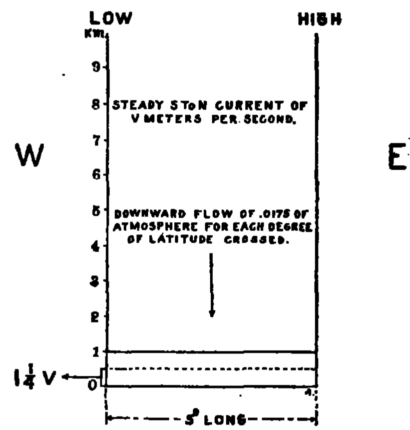

FIG. 1.—Cross section of 9 kilometers [1 km. to 10 km.] of a south to north current 5° wide, "maintained" in latitude 45°.

The representation is, moreover, borne out by the facts which are known as to the distribution of temperature in the atmosphere. For the 7 kilometers between the 1-km. level and the 8-km. level the temperature on the "high" side is "too warm," and therefore represents the effect of a downward flow while the pressure is maintained.[3] Hence it seems possible for the conditions for the maintenance of a south-to-north current to be realized in practice, though the adjustment would be delicate and might certainly be transient.

Motion from north to south.

Persistence in the reverse of the case just described, that is to say, in the case of a current flowing from north to south, is in one respect more difficult and in another more easy.

What we have to provide for here is not the thickening but the shrinkage of the current in consequence of the

[3] See the paper in the *Journal of the Scottish Meteorological Society* already referred to.

decrease of $\sin \lambda$ as successive circles are crossed. The numerical result applies equally, but in the opposite sense. Thus a current of velocity V flowing from north to south requires that air should be *fed* with an inflow which, if distributed over the whole side, would be $0{\cdot}0175 \frac{\cos^2\lambda}{\sin\lambda} lV$ at any level at which the wind velocity is V, in order to avoid fractional shrinkage of $0{\cdot}0175 \cot \lambda$ per degree of advance. It is more difficult to see how the air could be supplied; but the shrinkage of the current, while the distribution of pressure which controls it is maintained, presents little difficulty if the current in question may be supposed to remain an upper air current and therefore subject only to the pressure-distribution appropriate to the current. To explain the persistence of a current in the lower layers would make greater demands upon one's ingenuity, because the introduction of the necessary air would, as a rule, alter the distribution of pressure below, and limitations to prevent that alteration would have to be invented. Hence the maintenance of a current from north to south at all levels requires some artifice for the continuous production of the necessary pressure-distribution. The difficulty is further aggravated by the fact that, just as in the case of the south-to-north current, there is a flow-off from "high" to "low" in the surface layers; but unfortunately it flows *away from where it is required* to make up the loss due to change of latitude, and consequently that loss, as well as the loss by shrinkage, has to be made good if the northerly current is to be maintained.

Putting the two currents side by side as in figure 2, we see that the supply for the north-to-south current may possibly come from the surplus of the south-to-north current, but it can not be along the surface. It must be remembered that, so far as our information goes, we have no reason from observations for supposing that the relation between pressure and temperature in a northerly current is different from that in a southerly current, though the evidence is not quite conclusive, because the former has been less frequently the subject of investigation. The air supply ought, therefore, to be carried out in a similar manner in both cases. Persistence in this case, therefore, requires the surplus of the adjacent southerly current *and the outflow from the northerly itself* both to be delivered to the northerly current in the upper layers in order that the proper temperature distribution may be obtained.

Such a combination of circumstances may fairly be regarded as exceptional, and therefore the maintenance of a northerly current must be regarded as exceptional.

Changes from the steady state.

To complete the process of maintenance of the steady current from the north we should have to imagine the whole of the outflow in figure 2 toward the "low" from both sides conveyed to the upper part of the northerly current, and thus transferred from low pressure to high pressure as well as from low level to high level. It is possible to make out a process with the aid of the law of convection if the two currents are at different temperatures. In such a case the surfaces of equal pressure may be so sloped as to produce an apparent flow across isobars from low to high; but we have no such obvious and automatic explanation to give in the case of the northly current as in the case of the outflow of the southerly current. And, indeed, it was not intended to adduce the conditions for persistent maintenance with the object of claiming that they are generally satisfied in practice. On the contrary, the adjustment of the outflow in the southerly current to the conditions of persistence must be fortuitous and unlikely to be maintained for long; the adjustment of conditions for the maintenance of a northerly current is even more fortuitous. The reason for setting out the conditions of maintenance is rather to show that natural conditions of atmospheric currents are not, as a rule, those of persistence but of change. If the conditions of persistence which have been set out are not realised, the currents will change, and by Law 1 changes in currents imply changes in the distribution of pressure. Consequently, an atmospheric system which includes northerly or southerly currents has within itself elements and causes of change in the distribution of pressure. It is therefore unnecessary to attribute all changes to outside causes. It is preferable to consider the causes of the changes which are inherent in cases in which we cannot suppose the conditions of maintenance satisfied, and to regard external causes of change which are known to exist as supplementary.

It follows that we have not to regard a quiescent atmosphere all over the globe as the starting-point of our ex-

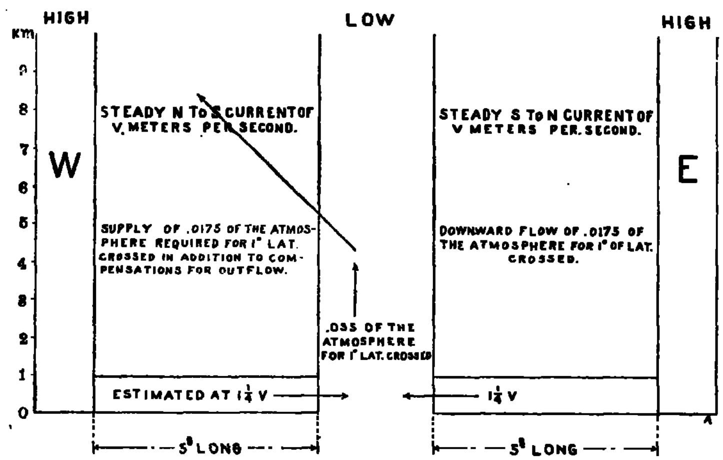

FIG. 2.—Cross section of 9 kilometers [1 km. to 10 km.] of two currents, south to north, and north to south, each 5° wide, "maintained" in latitude 45°.

planation of the present condition, but we have rather to regard the circumstances of transition from one set of conditions to another.

We may add some notes upon practical cases.

Persistent southerly current.

The maintenance of a southerly current has been shown to be a question of adjustment of velocities, and a southerly current lends itself comparatively easily to persistence. Examples of a persistent southerly current across the parallels of northern Europe furnish a well-recognized type of weather that seems to resist the incursions of cyclones from the west. A southerly current often extends throughout the vertical section of the atmosphere, as might be expected from the automatic thickening described above.

Persistent northerly current.

On the other hand, a northerly current requires constant reinforcement, and yet a northerly current, persistent for days over the northeastern Atlantic, is by no means unknown. It is possible that the necessary air in this case may be supplied by the gravitational flow of cold air off Greenland or northern Siberia, which must contribute a large amount of air to the surface layers above the northeastern Atlantic.

Replacement of a northeasterly current by a southwesterly current.

An example of the disturbance of persistence frequently occurs in the case of a northeasterly current with a southwesterly current above it, a case which is referred to in Mr. Cave's book on the *Structure of the Atmosphere in Clear Weather* as a frequent precursor of weather of the thunderstorm type, accompanied by the setting in of the southwesterly wind. The distribution of temperature is such as to change the direction of the pressure-gradient near the surface. Consequently the outflow from high to low goes from under the upper "low" to under the upper "high." The necessity for the thickening of the southerly current is therefore not relieved by the outflow, but accentuated thereby. At the same time the northeasterly current has to get thinner, so it is gradually replaced by the southwesterly current settling down to the surface. The appropriate redistribution of pressure at the surface accompanies the redistribution of air currents in the vertical section.

These examples are adduced because it seems not improbable that they give us the opportunity of watching the operation of the causes of change which are inherent in any actual state of atmospheric motion.

Let me summarize the attitude which seems to me to be appropriate for the meteorologist to take up in face of the complexities of the atmospheric circulation, by again referring to the position of the astronomer before the final enunciation of the laws of motion. Imagine the perplexity of the astronomer who, finding the heavenly bodies moving in all sorts of directions with all sorts of velocities, set himself to explain the motion which each possessed. To him the laws of motion bring the assurance that it is not necessary for him to explain why a body moves; it is the changes of motion which should occupy his attention. So the meteorologist, looking at the circulation of the atmosphere in obedience to the distribution of pressure, has not to ask himself why the pressure is high here or low there, but rather, "Is the distribution persistent, and if not, are the causes of change inherent in the existing circulation sufficient to account for the changes?" If it be said that, after all, the problem remains the same and the point of view is immaterial, it is right to remember that in astronomy the change in the point of view has simply reduced chaos to law.

From what has been already said, it appears that a steady state of persistent motion of the earth's atmosphere is in the highest degree improbable, because it can only occur in a combination of circumstances which are independently fortuitous; but it is desirable to call attention to a possible case of motion which is quasi-persistent in consequence of two concurrent and persistent infractions of the conditions of steadiness.

If we suppose the south-to-north and north-to-south currents of figure 2 placed back to back so as to form an anticyclonic section instead of the cyclonic section represented in figure 2, we find in juxtaposition a south-to-north current which must get rid of air, and a north-to-south current which must have air in order to maintain itself, and all that is required in order to maintain both currents is a transverse flow of $0{\cdot}0175\dfrac{\cos^2\lambda}{\sin\lambda}\,lV$ at any level where the current velocity is V from the south-to-north current to the north-to-south current. We can not accept this transverse motion as a part of steady motion, because the motion would not be strictly speaking along the isobars as prescribed by Law 1. But if we could persistently take the momentum necessary for the perturbation of the steady motion in compliance with Law 1 out of the general west-to-east circulation, we could have both the southerly and northerly currents maintained. It is not unreasonable to suppose that, as a westerly circulation has to be diverted northward to produce the northward circulation, the westerly momentum at the various levels may produce the effect described. In this case we should have the permanence of the anticyclonic distribution maintained by the persistent infraction of the law of relation of pressure to wind. At the same time a flow-off at the bottom outward in both cases has to be supplied, and in consequence there is a downward flow under permanent conditions of pressure over both sides of the ridge of "high" which would give the necessary warming of the air of a high-pressure region. Hence the case represented in figure 3 seems to furnish a possible example of a high-pressure region maintained in a quasi-steady condition by a transfer of air across the isobars in consequence of the uncompensated momentum; the flow-off on either

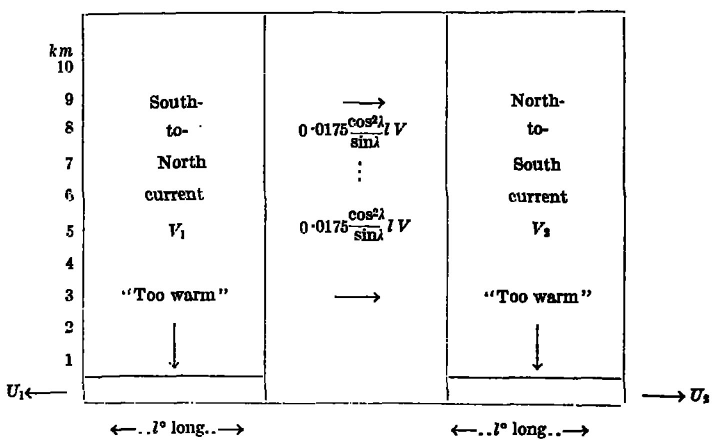

FIG. 3.—South-to-north current, V_1, supplying its own bottom outflow, U_1, and maintaining a parallel north-to-south current, V_2, and its bottom outflow, U_2, by transference of air across the "high" ridge.

side at the bottom from "high" to "low" denoted by U_1 and U_2 being provided by the adjustment of the currents V_1 and V_2.

Whether or not this be a true explanation, it certainly agrees with common experience in regarding a high-pressure area as more easily maintained persistently than a "low."

Propositions 2, 3, and 4.

These propositions, which deal with the application of the formula for change of pressure-difference with height (the unit of height being the meter), viz,

$$\frac{d\Delta p}{dh}=0.0342\frac{p}{\theta}\left(\frac{\Delta\theta}{\theta}-\frac{\Delta p}{p}\right),$$

to explain the dominance of the stratosphere and the lack of importance of the troposphere in the distribution of pressure at the surface, to compute the wind-velocity from the pressure-difference at any height and to explain the observed falling off of wind-velocity with height in the stratosphere, have been dealt with in the paper communicated to the Scottish Meteorological Society, and the work need not be repeated here, especially as Proposition 5 makes use of the same equations.

Proposition 5.—The calculation of the distribution of pressure and temperature in the upper air from the observations of structure represented by soundings with a pilot balloon.

A pilot balloon gives primarily the horizontal direction and velocity of the wind at successive heights, so that we may suppose that we have the horizontal direction and velocity of the wind at each kilometer as the data for the calculation.

The first step is to resolve the wind-velocity into two components, west to east and south to north.

By the application of Law 1 we can then compute the pressure-difference for 100 kilometers in the south-to-north direction and the west-to-east direction.

Thus, if Δp is the pressure-difference for a distance L taken along the direction of the wind velocity V, if θ, in absolute degrees, and p, in millibars, are the temperature and pressure, λ the latitude, ω the angular velocity of the earth's rotation, and R the constant of the characteristic equation for air, we have

$$V = \frac{R}{2\omega \sin \lambda} \frac{\theta}{p} \frac{\Delta p}{L} = K \frac{\theta}{p} \frac{\Delta p}{L}.$$

And since both velocity and pressure-difference, or gradient, are vector quantities, we get for the northward and westward components of the pressure-gradient per hundred kilometers

$$\Delta_N p = \frac{1}{K} \frac{p}{\theta} \text{ (W to E)}$$

and

$$\Delta_W p = \frac{1}{K} \frac{p}{\theta} \text{ (S to N)},$$

where (W to E) and (S to N) indicate the components of the wind-velocity resolved in those two directions.

Now from a pilot balloon ascent we can not get the value of p/θ for the special occasion of the ascent, but there is really little variation from time to time of this ratio. For the greater part of the troposphere variations of pressure and temperature go together, and the whole range of variation of θ for any particular time of year is less than 10 per cent, and the whole range of variation of p is of the same order. Consequently a mean value of p/θ may be taken as a first approximation for the purposes of the calculation.

The following is a table of average values of p/θ:

TABLE 2.—*Table for values of p/θ at different levels—average of results in "Geophysical Journal," 1912.*

Height.	p/θ.	Height.	p/θ.	Height.	p/θ.	Height.	p/θ.
Km.		*Km.*		*Km.*		*Km.*	
20	0·26	15	0·53	10	1·18	5	2·11
19	·28	14	·64	9	1·35	4	2·35
18	·32	13	·75	8	1·52	3	2·61
17	·39	12	·87	7	1·70	2	2·91
16	·46	11	1·02	6	1·90	1	3·24
						Gd.	3·55

Having thus computed the pressure-difference for 100 kilometers, in two directions at right angles, for the level of each kilometer, we may next obtain by subtraction the change of pressure-difference for each kilometer. The use of the mean value for p/θ will not altogether invalidate the process, because the variation from kilometer to kilometer depends generally on the ordinary diminution of pressure with height rather than on any extraordinary distribution of temperature.

Substituting the value of the rate of increase of pressure-difference per kilometer of height in the equation

$$\frac{d\Delta p}{dh} = 34.2 \frac{p}{\theta}\left(\frac{\Delta\theta}{\theta} - \frac{\Delta p}{p}\right)$$

and again assuming a value of θ/p, we can compute $\Delta\theta$ provided we have a value of θ which can properly be substituted in the equation.

Here, again, we must have recourse to the mean value, as we have no observation of actual temperature at the time; but, again, the error made is not fatal to the practical success of the calculation, because θ comes in as a factor which affects the scale of the variation; it does not affect the sign. By taking the mean value for the month instead of the actual value the error is probably less than 10 per cent and the whole error of employing mean values for actual values probably amounts to less than 20 per cent; and in considering the distribution of pressure and temperature in the upper air we are not yet in a position to reject observations and information which may be in error by as much as a fifth.

Consequently we may properly use the calculation here indicated to give at least a rough but working idea of the distribution of pressure and temperature at successive levels in the atmosphere when we know the velocity and direction of the wind there.

The errors in p/θ and θ are less important in considering the nature of the distribution, because the same values, right or wrong, are used for both components at the same level.

The following table of monthly averages gives values which may be used in the absence of any special information for the particular occasion:

TABLE 3.—*Average temperature (°A.) at different levels for months.*

1. FOR BRITISH ISLES. TAKEN FROM "GEOPHYSICAL MEMOIRS," NO. 2 (W. H. DINES).

Height.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Km.	°	°	°	°	°	°	°	°	°	°	°	°
14	216	217	219	221	222	223	222	221	219	217	216	215
13	216	217	219	221	222	223	223	221	219	218	217	216
12	217	218	219	220	221	222	222	221	221	219	218	217
11	217	217	217	219	220	221	222	222	221	220	219	218
10	220	220	220	222	224	225	226	226	226	224	223	221
9	224	223	224	226	229	231	234	233	233	231	228	225
8	230	229	230	232	236	238	241	241	241	238	235	332*
7	237	236	237	239	242	245	247	248	247	245	241	238
6	243	243	244	246	249	252	255	255	254	251	249	245
5	250	249	250	252	256	259	261	262	261	258	255	252
4	257	256	257	259	262	265	267	268	267	264	261	258
3	263	262	263	265	268	271	273	274	273	270	267	264
2	267	266	267	270	273	276	278	279	278	275	272	269
1	271	271	273	276	279	282	283	283	281	279	275	272
Gd.	276	276	277	282	285	288	289	289	286	283	280	277

*232°A?—C. A.

I give in Table 3 a specimen of the calculation as applied to the results of a sounding with a pilot balloon on April 29, 1908.

TABLE 4.—*Computation of pressure distribution and temperature distribution from a pilot balloon ascent of April 29, 1908.*

Height.	Velocity.	Direction.	W to E Comp.	$\frac{W \text{ to } E}{K}$ $K=25.4$.	$\frac{p}{\theta}$.	$\frac{W \text{ to } E}{K} \times \frac{p}{\theta} = \Delta_N p$.	Change per km. (increase).	$\frac{\text{Incr. per km.}}{34.2}$	θ.	$\frac{\text{Incr. per km.}}{34.2} \times \theta$.	$\frac{\text{Incr. per km.}}{34.2} \times \theta + \Delta_N p$.	$\left(\frac{\text{Incr. per km.}}{34.2} \times \theta + \Delta_N p\right) \div p/\theta$.	$\frac{1}{\Delta_N \theta} \times 100$.
km.	*m/s.*	*°Az.*	*m/s.*			*mb.*	*mb.*		*°A.*				*km.*
6	20.5	300	+17.75	+0.70	1.88	+1.32	+0.26	+0.0073	248	+1.81	+3.13	+1.66	+ 60
5	15.0	300	+12.99	+ .51	2.08	+1.06	+ .29	+ .0085	254	+2.16	+3.22	+1.55	+ 64
4	8.5	280	+ 8.37	+ .33	2.33	+0.77	+ .12	+ .0035	261	+0.91	+1.68	+0.74	+135
3	6.5	265	+ 6.48	+ .25	2.58	+ .65	− .22	− .0064	267	−1.71	−1.06	− .41	−244
2	8.0	250	+ 7.52	+ .30	2.90	+ .87	+ .33	+ .0091	272	+2.48	+3.35	+1.08	+ 93
1	5.0	240	+ 4.33	+ .17	3.18	+ .54	+ .08	+ .0023	278	+ .64	+1.18	+ .37	+270
0	5.0	220	+ 3.21	+ .13	3.50	+ .46							
Height.	**Velocity.**	**Direction.**	**S to N Comp.**	$\frac{S \text{ to } N}{K}$ $K=25.4$.	$\frac{p}{\theta}$.	$\frac{S \text{ to } N}{K} \times \frac{p}{\theta} = \Delta_W p$.	**Change per km. (increase).**	$\frac{\text{Incr. per km.}}{34.2}$	θ.	$\frac{\text{Incr. per k.}}{34.2} \times \theta$.	$\frac{\text{Incr. per km.}}{34.2} \times \theta + \Delta_W p$.	$\left(\frac{\text{Incr. per km.}}{34.2} \times \theta + \Delta_W p\right) \div p/\theta$.	$\frac{1}{\Delta_W \theta} \times 100$.
km.	*m/s.*	*°Az.*	*m/s.*			*mb.*	*mb.*		*°A.*				*km.*
6	20.5	300	−10.25	−0.41	1.88	−0.77	−0.15	−0.0044	248	−1.09	−1.86	−0.98	−102
5	15.0	300	− 7.50	− .30	2.08	− .62	− .48	− .0140	254	−3.56	−4.18	−2.01	− 50
4	8.5	280	− 1.47	− .06	2.33	− .14	− .19	− .0056	261	−1.47	−1.61	− .69	−132
3	6.5	265	+ 0.57	+ .02	2.58	+ .05	− .27	− .0079	267	−2.11	−2.06	− .80	−125
2	8.0	250	+ 2.74	+ .11	2.90	+ .32	0	0	272	0	+ .32	+ .11	+909
1	5.0	240	+ 2.50	+ .10	3.18	+ .32	− .21	− .0061	278	−1.70	−1.38	− .45	−222
0	5.0	220	+ 3.84	+ .15	3.50	+ .53							

I have used this method for the calculation of the distribution of pressure and temperature in the cases represented by photographs of models in Mr. C. J. P. Cave's book on the *Structure of the Atmosphere in Clear Weather*,[4] which includes that given in detail above. Some of the results are given below—the problem being understood to be stated thus: *Given the wind velocity at any point, to find coordinates for drawing the isobar for the next higher millibar and the isotherm for the next higher degree of temperature.* It will be remembered that the isobar over the point of observation itself is to be taken parallel to the wind direction in accordance with Law 1, and the direction of the isothermal lines will be taken parallel to the line joining the computed coordinates, so that the distribution of pressure and temperature is to be represented each by two parallel lines, the coordinates giving their direction and their distance apart.

1. SOUNDING OF MAY 5, 1909, 6h 43m P. M.

"*Solid current*": *Wind approximately uniform in direction and velocity from 2 kilometers to 10 kilometers.*

TABLE 5.

Height.	Distance of next higher isobar.		Distance of next higher isotherm.	
km.	*km.*	*km.*	*km.*	*km.*
9–10	143 N	233 E	93 N	93 W
8–9	143 N	181 E	1000 N	1250 E
7–8	123 N	291 E	454 S	54 E
6–7	114 N	292 E	137 N	74 W
5–6	99 N	141 E	100 S	139 W
4–5	77 N	110 E	832 N	58 E
3–4	67 N	187 E	303 S	909 W
2–3	58 N	144 E	769 N	196 W
1–2	54 N	353 E	270 N	49 E
0–1	...	...	...	...

In this case it is interesting first to notice the gradual separation of the isobars with increasing height and consequently diminishing density. This is the ordinary condition for the velocity remaining invariable with height.

[4] Cambridge University Press, 1912.

Secondly, it is noteworthy that the separation of the isotherms is generally large and also very irregular, showing approximate equality of temperature in any layer, but great want of conformity between one layer and another. Such variations in the distribution of temperature may easily be accounted for by local convection producing changes of temperature and possibly clouds, and it leads us to reflect that the convection which produces local clouds will also produce local modifications of temperature and consequently local modifications of pressure and wind velocity. If we ask whether such local variations of temperature and wind are at all probable, we have only to refer to the records of the ascents of registering balloons and of anemometers, or of pilot balloon ascents, to give an affirmative answer.

Nothing is more noteworthy than the irregular variations in temperature difference as given by a pair of soundings with registering balloons, and the curious local irregularities of wind disclosed by pilot balloon ascents. Hitherto it has been customary, on quite general grounds, to regard them both as possibly due to the uncertainties of observation. We now see that they may equally well be important evidence of complication in the structure of the atmosphere.

Those whose temperament inclines them that way have still the possibility of uncertainties in observation to fall back upon; but the better plan would seem to be to arrange for simultaneous ascents of registering balloons and pilot balloons, so that the actual and computed distribution of temperature may be compared. The interesting feature of the comparison would be that, if the method of computation here indicated (with its acknowledged uncertainties in taking mean values for p/θ and θ instead of actual values) should prove serviceable, then one pilot balloon ascent gives for practical purposes almost as much information as three registering balloons.

Apart from the uncertainties which have been mentioned, the conclusions as to the distribution of temperature and pressure are incontrovertible by those who accept Law 1, and *per contra* if the conclusions are sustained Law 1 receives its most complete vindication.

2. SOUNDING OF SEPTEMBER 1, 1907.

Westerly wind rapidly increasing aloft.

TABLE 6.

Height.	Distance of next higher isobar.		Distance of next higher isotherm.	
km.	km.	km.	km.	km.
4	68 S	∞ E or W	86 S	119 E
3	77 S	400 W	44 S	555 E
2	139 S	294 W	119 S	185 W
1	196 S	526 W	43 S	80 E

The increase in the intensity of the pressure distribution with height is clearly shown and finds its explanation in a steep temperature gradient from south to north.

3. SOUNDING OF NOVEMBER 6, 1908, 10^h 55^m A. M.

Reversal of direction from E.S.E. in the lowest three kilometers to W.N.W. in the reach from 4 kilometers to 9 kilometers.

TABLE 7.

Height.	Distance of next higher isobar.		Distance of next higher isotherm.	
km.	km.	km.	km.	km.
8-9	185 S	356 W	96 S	312 W
7-8	204 S	356 W	416 S	770 W
6-7	200 S	416 W	294 S	189 W
5-6	233 S	435 W	139 S	625 E
4-5	344 S	665 W	101 S	109 W
3-4	5,000 S	4,000 E	119 S	270 W
2-3	588 N	416 E	286 S	108 W
1-2	100 N	142 E	24 S	65 W
0-1	77 N	172 E	34 N	40 E

The gradual diminution of velocity up to 4 kilometers where the isobars become very wide apart, is well marked in the second and third columns; and it is seen that the reversal is to be accounted for by a rapid rise of temperature to the southwest in the second and third kilometers, with a similar distribution of temperature of less marked character in the higher layers.

It will be noticed that in the second and third kilometers, where the reversal is determined, the slope of temperature is opposite to the slope of pressure, a condition which we have already noticed as being characteristic of large change of pressure-difference with height. In the sixth kilometer the next higher isotherm is found a long way off on the east instead of on the west, as in the layers above and below. The change is not really very large, as the temperature conditions are nearly uniform in that region as regards the west-to-east direction, but it furnishes a reminder of the close association which we must expect to find between slight changes in temperature distribution and in the direction and force of the wind.

4. SOUNDING OF APRIL 29, 1908.

Northwesterly current in the upper layers crossing a lower current from the southwest.

This is the example of which the details of the working are shown in Table 4, and it is one of great interest, because it is characteristic of the advance of a well-developed cyclonic depression from the westward. It has long been recognized, by seamen and other observers of weather, in observations of upper clouds which are seen to be moving from the northwest while the surface winds are coming from the southwest. It is one of the surest signs of the rainfall which occurs in the front of a cyclonic depression. The table already given shows the values of $\Delta_N p$ and $\Delta_W p$ for each kilometer level, and the values of $\Delta_N \theta$ and $\Delta_W \theta$ computed from the changes in the pressure-differences for successive kilometer steps.

We may note here an ambiguity of notation, which we ought to find some means to remove, and which ought at least to be made clear. In the table [Table 4] Δp and $\Delta\theta$ are used to indicate the *slope* of pressure and of temperature in the two directions N. and W. Thus in the table, when Δp or $\Delta\theta$ is positive for a given direction, it is to be understood that it represents the *fall* of pressure in that direction. But the usual convention of the differential calculus is that an *increase* in the quantity represented is indicated by a positive value of the difference. The ambiguity arises from the use of the convenient symbol Δ to denote the difference, while the meteorological practice is to think of gradient as represented by downward slope. I have not found any convenient new symbol to use instead of Δ to indicate a negative difference, so the ambiguity remains for the present, though I feel that an apology is due for it.

In order to present in a table the corresponding values of Δp and $\Delta\theta$ for the same level, I have taken the means of the two values of Δp for the top and bottom of the kilometer to which $\Delta\theta$ refers. This practice is, perhaps, rather doubtful, but except in Table 6 it has been followed in the tables already given, so I adhere to it in this one.

Converting by simple inversion the figures for Δp and $\Delta\theta$ per 100 kilometers into distances along the axis of the intercepts of the next higher isobar and isotherm, respectively, we obtain the following:

TABLE 8.

Height.	Distance of next higher isobar.		Distance of next higher isotherm.	
km.	km.	km.	km.	km.
5-6	84 S	143 W	60 S	102 W
4-5	109 S	263 W	64 S	50 W
3-4	141 S	2,000 W	135 S	132 W
2-3	131 S	526 E	244 N	125 W
1-2	141 S	312 E	93 S	909 E
0-1	200 S	232 E	270 S	222 W

In this table the gradual conversion of a southerly component into a northerly component associated with higher temperature to the westward is very noticeable.

It will be seen that the isobars above 4 kilometers are, roughly speaking, at right angles to those in the lowest kilometer, which is, of course, in accordance with the wind observations; but that the isotherms, with some fluctuations, particularly in the second kilometer, are similarly arranged at the top and at the bottom. That is to say, the upper winds are flowing from the northwest with the higher temperature on the southwest side, while the lower winds are moving transversely from the southwest with a distribution of temperature parallel to that of the upper air, but in this case the isotherms are across the wind.

These results are represented in figure 4, which was originally drawn to the same horizontal scale as the larger chart of the Daily Weather Report, and it is clear that in the lowest stage the columns of warmer air brought in by the southwesterly current are being carried underneath the parallel columns of the upper current. Up to 4 km., where the wind has become westerly, we have a distribution which produces the same effect. The wind is always carrying warmer air under colder air, and as, by Proposition 1, a southerly current tends to thicken and a northerly current to give way, the pushing under of the warmer air becomes more effective, until instability occurs and rainfall sets in. The irregularities which are

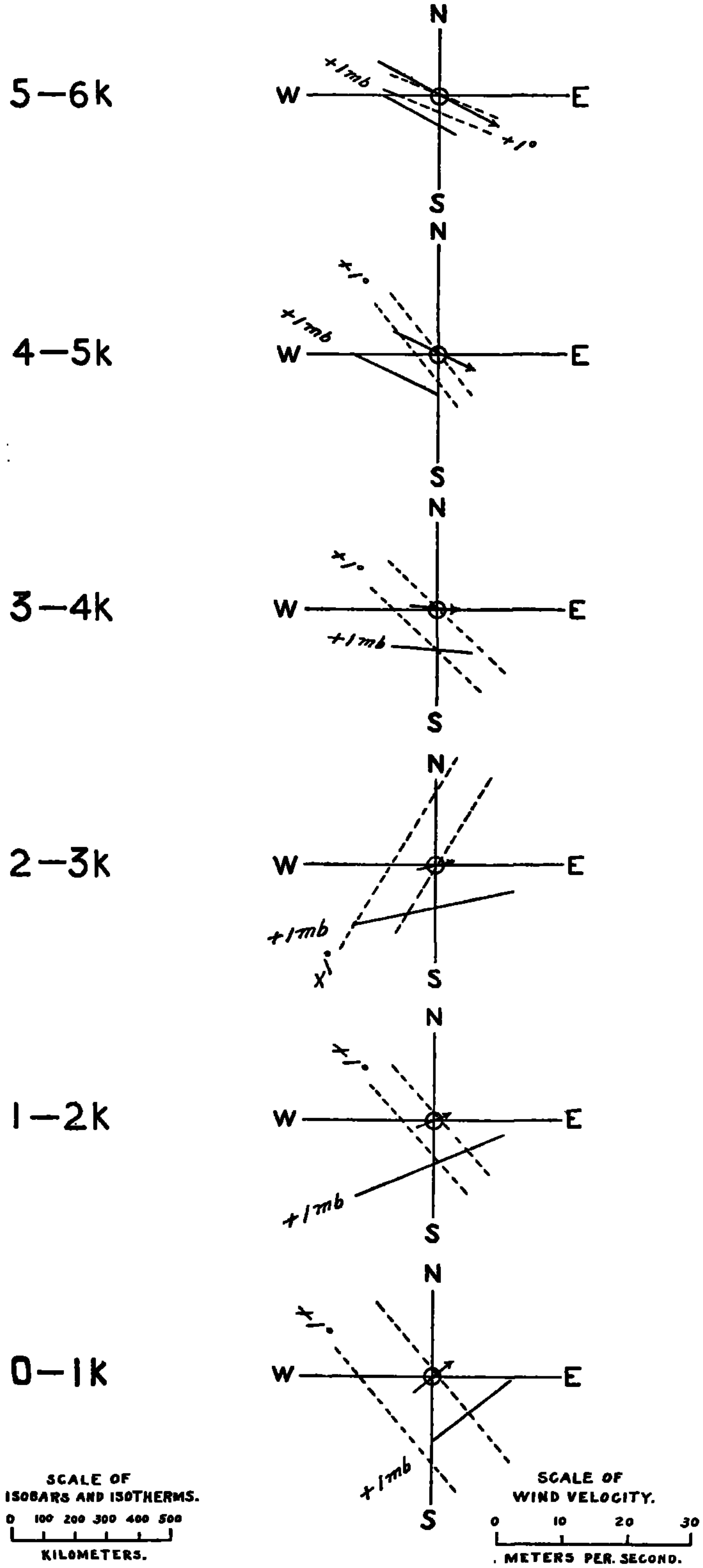

FIG. 4.—Pilot balloon sounding, April 29, 1908. Northwest wind over southwest: characteristic of an advancing depression. The arrow shows the direction and velocity of the wind; the full line, the position of isobar *next above* that which passes through the station. The dotted line through O shows the isotherm passing through the station; the parallel dotted line, the isotherm for *one degree higher* than that of the station.

shown in the distribution of temperature are probably due to previous convection.

We have here, therefore, the assurance of rainfall conditions as the southwesterly wind pursues its course under the northwesterly in front of the approaching depression. The rainy condition of that part of a depression is thus directly accounted for.

The characteristic rainfall of a cyclonic depression is generally associated with a general convergence of the surface isobars, but this hypothesis is difficult to follow into details, because the convergence is general over the area, while the rainfall is local. The analysis of the conditions of the upper air here set out shows that there is good reason for rainfall in the upper layers, to which the doctrine of general convergence can not safely be held to apply.

To the examples which are taken from Mr. Cave's work, I may add one for October 16, 1913, which was reported to me by Mr. J. S. Dines in connection with his work for the branch meteorological office at South Farnborough.

On that day, at Pyrton Hill, where the sounding was made, there was a sudden change of wind between 1,100 and 1,500 meters height from a reasonably steady wind from nearly due south into one almost as steady from due north, the change being accomplished within half a kilometer. The analysis in this case shows for the layer between 500 and 1,100 meters a temperature distribution in isotherms nearly north and south with the warmer air on the east, and above 1,500 meters an entirely different distribution with isotherms nearly east and west, and cold to the northward. The intermediate layer, 400 kilometers thick, showed a very rapid increase of temperature to the west—as much as 7° C. per hundred kilometers.

The complete arrest of the northerly current and production of a calm by the annihilation of the gradient between 1,100 and 1,500 meters is very remarkable, but nevertheless a real fact. The accompanying temperature difference is probably due to a strong temperature "inversion" at a height of about 1,500 meters at the place of observation and of 1,100 meters at a place 100 kilometers distant to the west. On that occasion it lasted for some time, as it was found an hour afterwards by a second balloon; but it must be remembered that it was a region of no velocity, and therefore the relatively warm and cold airs were not moving. In order to get them away, either convection must take place or a gradient must be created.

Proposition 6.—The general circulation of the atmosphere in the Northern Hemisphere.

The reasoning in this proposition is more general in form than that of the foregoing propositions. The extension of our knowledge tends more and more to strengthen the conclusion that the proximate cause of the variations of pressure in the region of the British Isles must be looked for in the layer at a height of about 7 to 9 kilometers; it is the layer of maximum wind velocity just under the stratosphere, and it is also the layer within which must be located a rapid transition of slope of temperature. Below it, as set out in Lemma I the slope of temperature follows the slope of pressure; above it, the slope is in the opposite sense. The mechanism by which the changes of pressure are produced is unknown; but this much is apparently true, that within the layer referred to, the relation between the pressure and temperature of the air at two places on the same level is that of adiabatic expansion. Above the critical layer where this relation holds, the air in the high-pressure area is "too cold," and below it, for 5 or 6 kilometers at least, it is "too warm." [5] We may suppose that air becomes "too warm" by the dynamical warming of downward convection, and, perhaps, also that it becomes "too cold" by piling up under the stratosphere and readjustment of the several layers within the stratosphere, so that pressure on the sample which causes the bulging is reduced, while that over the surrounding regions is increased.[6] Radiation is left out of account—whether rightly or wrongly, it is not possible at this stage to say.

The motion of the critical layer is on the average from west to east, but not invariably so, and apparently the temperature relations which have been described are not dependent upon wind direction. Other phenomena, so far as they have been observed, seem to indicate a similar symmetry, but there is no sufficient evidence for supposing that the phenomena are necessarily centered locally. In fact, according to the distribution of isobars at 4 kilometers computed by Teisserenc de Bort (Lemma II), the average motion does not differ much from a circulation round the pole which, once set up, might be persistent with little change if it was everywhere adjusted to the barometric gradient. The actual motion, however, certainly does change, and is, in fact, constantly changing.

Let us consider the conditions of Teisserenc de Bort's average isobars and the forces which are available to produce the perturbations of a supposed original circumpolar circulation indicated thereby. I have already remarked that, for such a circulation as that represented by Teisserenc de Bort, the isobars for 4 kilometers may fairly be accepted as applicable at 7 kilometers also, because the changes of pressure difference between 4 kilometers and 7 kilometers are in ordinary circumstances very slight.

Taking the average map for January, it will be noticed that the isobars at 4 kilometers are clearly not circles round the pole. If they were so, a steady circulation would be a natural conclusion. It has been already postulated in Lemma II that they are in reality indented ovals or approximate figures of eight with the lobes over the Asiatic and American Continents and the inward bends over the two oceans. I purpose considering, first, the effect of convection as a possible cause of the deviation from the circular shape. The shape which we have to explain is exactly opposite of that which is often shown on synchronous charts of the distribution of pressure at the *surface* of the Northern Hemisphere in winter, and which has "highs" over the continents and "lows" over the oceans. I remark in the first place that, to derive the figure-of-eight shape from the circular shape, one can not rely simply upon the nutation of a west-to-east circulation round the pole; one must superpose either a pair of anticyclonic systems, elongated north or south, over the oceans, or a pair of cyclonic systems over the continents, of which we can at present only determine the southern portions; or we might arrive at the actual shapes by adjustments of both kinds. If we assumed positions for the original circular isobars, it would be a simple matter to give numerical values of the superposed anticyclones or cyclones. But the circumpolar circular isobars are hypothetical, and, at the present stage, the numerical work indicated would be unremunerative. Let us assume, however, such an initial circumpolar system, and consider the physical forces which would disturb its motion.

[5] See Journal Scottish Met. Soc., 1913, loc. cit.

[6] See a note on the Perturbations of the Stratosphere in Publication 202 of the Meteorological Office.

The only force immediately at hand is that of gravity, due indirectly to the cooling of the surface air on the land and frozen sea in the Arctic night operating in accordance with Law 3, the law of convection. This may produce a real effect of some magnitude on land slopes. It is not, I think, necessarily effective over level surfaces, because there is no slope down which the cooled air can flow.

I have always hesitated about the common explanation of the trade winds and other well-known phenomena based upon the reverse process of surface heating. Surface heating and surface cooling necessarily produce a certain amount of expansion and contraction, but not necessarily any continuous convection current. Convection requires the juxtaposition of warm air and cold air, and, if the region is big enough, the result of surface heating may easily give rise to a heated volume of air surrounded by isobars and air currents that prevent any continuous process of general convection. Local convection there would be, but that need only extend high enough up to take up the day's heat. All the main air currents of the globe have pressure distributions to guide them. They can not usefully be called convection currents.

So, if we had, say, a million square miles of level ice round the pole, I can not see that the cooling of that area need produce any considerable effect upon the distribution of pressure; but if the cooling takes place on slopes, we at once get the force of gravity to help, and one can no more suppose the downward flow of the air to be stopped than the flow of a river to be permanently arrested. Hence there must be in winter a continual flow of air off the great land areas of the Northern Hemisphere if they have any slope. The air fall off Greenland, for example, must be enormous. Every description by explorers in the Antarctic seems to support the suggestion of a great cold-air cascade from the Antarctic continent. How much flows, and where it flows to, I can not say; ultimately it must find its way to warmer latitudes by some route or other; but these air flows must be a real cause of alteration in the distribution of pressure, and it is to the land slopes which are losing heat that we may trace an indubitable influence, and therefore a disturbance of the uniformity of circulation. Apart from compensation, a flow-off of 1 meter thickness of air would mean a reduction of pressure of 0.1 millibar.

The facts which are here represented are sometimes taken as indicating the formation of anticyclones over the Arctic and Antarctic land areas. When those areas are represented by plateaus 10,000 or 15,000 feet in height, the surface anticyclone may become merely a hypothetical construction supposed to occupy the space which is really occupied by land and not by air at all. To a considerable extent the great Asiatic and American anticyclones depend upon the reduction of observations to sea level under conditions which can have no real existence. The mountain slope might possibly operate, in the maintenance of a cyclonic circulation in the upper air, much like the hole in the bottom of a basin, and the actual land surface at the high level might therefore be a region of cyclonic circulation.

Similar phenomena must of course happen locally, and they are well known in mountainous regions, though we can hardly expect the smaller local examples to show much effect in the distribution of pressure over the globe.

But we may assume that cold land slopes in winter are the cause of a constant abstraction of air from the lowest layers of the atmosphere in those regions. The cold air flows away by gravity, and since the *surface pressure* is apparently still maintained, the efforts to redress the loss of air have to be carried out in the upper atmosphere and in accordance with its laws; consequently we should expect to find a cyclonic circulation in the level in which the replacement is taking place. The cyclonic circulation may operate to prevent the pressure being made up overhead, but it can not prevent the cold air from flowing downhill unless the reduction of pressure is enough to reduce the density by as much as the low temperature increases it, and this is a difficult task, for near sea level it takes more than 3 millibars loss of pressure to make up for a single degree loss of temperature.

Hence we may suppose that the constant drainage of the land areas would result in the superposition of a cyclonic distribution at high level over them, and the continental lobes of Teisserenc de Bort's isobars for the upper air may well be due to this cause.

But the cause is obviously a very variable one, depending upon the distribution of cloud and other circumstances. Statistically, its effect upon the circulation of the upper air is to exaggerate the pressure gradient for westerly winds over the Temperate Zones of the continents, and to diminish the gradient northward. Thereby we introduce into the circulation local accentuation of current, which must be disposed of by some dynamical process.

The next step in the consideration rests upon the fact that by superposing a cyclonic depression upon the circumpolar circulation we displace a part of that circulation to the southward and reduce the northern part. Taking the case of Teisserenc de Bort's map for January, the westerly run of isobars over America and Asia is about 10° to 20° of latitude lower than over the oceans, and these two positions of westerly circulation have to be connected by isobars which cross the parallels of latitude, and therefore have a south-to-north and a north-to-south component respectively. Therefore, they can only be maintained persistently under the conditions set out in Proposition 1. Now, it has been shown in the discussion of Proposition 1 that permanence of a quasi-steady character might be realized in the case of an anticyclonic ridge having a south-to-north current on its western side, and *vice versa*, provided that momentum was being taken out of the westerly circulation in order to provide a slight eastward deviation from the isobars setting to the north. Such a case would be fairly represented by the deviation from circular isobars shown over the oceans on Teisserenc de Bort's map for January, and hence the form of those isobars may be arrived at by the influence of a steady flow-off of air down the land slope of the Arctic regions and the steady deviation of the wind from the direction of the southwest to northwest isobars on the western sides of the oceans in consequence of the momentum of the westerly circulation.

Meanwhile, what happens to the cold air which has run off the land areas? That has to be steered about by the distribution of pressure in the upper air as modified by any special peculiarities of temperature in the lower regions, and all sorts of complications may arise from this cause. So far as it goes, its density tends to set up high pressure over the regions which it covers, and so to make a slope of pressure southward and cause easterly winds on its southern side. Whenever in a mass of air temperature-fall is in the opposite direction to pressure-fall, great change in the horizontal distribution of pressure underneath is the result, and many of our local variations of pressure may fairly be attributed to the reactions which these cold masses of air offer to the attempt (in the end futile) on the part of the upper air to steer them round the pole from west to east. By their eastward motion these masses of cold air are always reminding us that if left to themselves, without the overpowering guidance of the pressure distribution of the upper air, they would form a circulation round the pole in opposition to the circulation of the upper air, with which they are in perpetual conflict.

TURBULENT MOTION.

In the study which has been the subject of the foregoing pages we have always considered the motion of the air to be regulated by a distribution of pressure balanced by the rotation of the earth, except in regard to the surface layer and one other suggested exception when the momentum of the general westerly circulation was invoked. It should here be noted that by this limitation to what may perhaps be called "great circle motion," we are considering almost exclusively the circulation above that half of the earth's surface which is north of the northern tropic and south of the southern one. There is another section of meteorology which has to deal particularly with the region between the Tropics, where the beginnings of tropical revolving storms are to be found. These storms, which have a diameter of some hundred miles or more, as well as the tornadoes which have a diameter of perhaps a quarter of a mile, belong to the subject of turbulent motion, with which the eddies and whirls that are produced by obstacles on the surface of the ground are also associated. All these phenomena of turbulent motion, important as they sometimes are in real life and death, must be treated in a manner different from that of the present communication.

BIRKELAND'S THEORY OF THE ZODIACAL LIGHT.[1]

[Dated Weather Bureau, Washington, D. C., May 1, 1914.]

Birkeland finds that several of his experiments [2] with a magnetized, phosphorescent terrella in a large vacuum chamber, furnish phenomena which serve him as a starting point for an explanation of the zodiacal light and the gegenschein.

The position of the zodiacal light has now been definitely shown to be closely related to the position of the solar equator, rising and sinking with it, and is not so immediately related to the ecliptic as former general opinion held it to be. One of the most significant, and heretofore unexplained, characteristics of the zodiacal light is the pulsatory character of the variations in its brightness or intensity, and in its shape. These pulsatory changes appear to an observer to be akin to those shown by the aurora and by terrestrial magnetism, and have been correlated with pulsatory oscillations in the terrestrial magnetic field. There is no lack of impeccable observations and records of this pulsation in the zodiacal light, witness writings by Humboldt, Birt of Kew, George Jones of the United States Exploring Expedition to Japan, and Birkeland at Halde, Kaafiord, and Khartum. Evidently an adequate theory of the zodiacal light must account for this feature of it. Birkeland therefore thinks "it very probable * * * that the zodiacal light must be primarily occasioned by electrical phenomena."

Birkeland regards the sun as a great magnet, having a "magnetic moment of the order 10^{28} or about 150 times as great as that of the earth," and that its magnetic equator is essentially coincident with its heliographic equator. Further he finds no good reason for supposing that the sun's magnetic axis is not coincident with its axis of rotation.

[1] The Norwegian Aurora Polaris Expedition, 1902-1903. V. 1, sec. 2, chap. 5. Christiania. 1913. f°.

[2] Described in "The origin of worlds." By Prof. K. Birkeland, Sci. Amer. suppl., Nos. 1957, 1958. New York, July 5, 12, 1913.